BU-HEP 96-15

Transport Phenomena and Electroweak Baryogenesis in the Two-doublet Higgs Model

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Abstract

We use transport equations to compute the evolution of the quark plasma at the electroweak phase transition in the two-doublet Higgs model, obtaining in a more rigorous and quantitatively accurate way results consistent with previous work. We discuss the model in connection with the electroweak baryogenesis scenario, and claim that the observed baryon asymmetry of the universe can be obtained within this model with a suitable choice of parameters.

1 Introduction

Electroweak baryogenesis was proposed years ago [1] as a possible mechanism to generate the estimated baryon asymmetry of the universe ($n_B/s \sim 10^{-10}$), and it has been the subject of a great amount of work ever since [2]. Although the Standard Model at the electroweak phase transition satisfies the three necessary conditions pointed out by Sakharov [3], a quantitative analysis is far from being trivial. In principle, the actual computation of the final baryon asymmetry requires a precise knowledge of the parameters (including those of the Higgs sector), of the details of the phase transition, of the B-violation rate near and at the phase transition, and of the transport properties of the QCD plasma in a non-uniform background field, which we do not yet have. A few general and persuasive conclusions have nonetheless been reached, and there is some consensus on the broad lines of this scenario:

- At temperatures higher than the EW critical temperature, B violating processes [4] are fast, and can wash out any previously created B asymmetry. Soon below the critical temperature, B violation is heavily suppressed, and can be safely neglected, implying that the baryons we see now must have been created at the EW phase transition. This argument assumes the absence of any primordial net $B-L$ number, which no EW process (including the anomalous ones) can wipe out and which would still be observable today, without any need for baryon production at the weak scale. However, in the absence of a compelling model predicting such a $B-L$ asymmetry, we are going to ignore this second scenario.

- A sufficiently strongly first-order PT is required, to prevent the newly created baryon asymmetry from being immediately washed out by the still strong B-violating processes.

- During nucleation, the expanding bubbles provide the necessary violation of CP and (macroscopic) CPT, thereby creating a net amount of CP-odd fields. This process takes place at the bubble walls, where the variation in the background fields is most rapid.

- The actual B generation takes place when the CP-odd fields reach *outside* the bubble, where B-violating processes are fast. For this reason, it is important to understand the transport properties of those fields.

- In any case, the Minimal Standard Model is quantitatively inadequate because it does not contain enough CP violation [5], so that extensions to the MSM have to be considered, and can be constrained (albeit loosely) by this cosmological test.

In this paper, we focus on the transport properties of the plasma at the EW phase transition in the two-doublet Higgs model and discuss their effect on baryogenesis. In the next section we introduce the model and discuss its qualitative properties near the phase transition. In section 3 we show how to derive kinetic equations from microscopic principles (a result already present in the literature [6]), and apply them to the problem at hand. Section 4 contains the explicit computation of the several parameters in the kinetic equations, and in section 5 we discuss our results and compare them with previous work.

2 The model

As mentioned above, we need to extend the Minimal Standard Model to include more CP-violating terms. Our choice is to introduce a second Higgs doublet, with the following potential [7]:

$$\begin{aligned} V(\phi_1,\phi_2) &= \lambda_1(\phi_1^\dagger\phi_1 - v_1^2)^2 + \lambda_2(\phi_2^\dagger\phi_2 - v_2^2)^2 + \lambda_3[(\phi_1^\dagger\phi_1 - v_1^2) \\ &+ (\phi_2^\dagger\phi_2 - v_2^2)]^2 + \lambda_4[(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)] \\ &+ \lambda_5[Re(\phi_1^\dagger\phi_2) - v_1 v_2\cos\xi]^2 + \lambda_6[Im(\phi_1^\dagger\phi_2) - v_1 v_2 \sin\xi]^2 \end{aligned} \tag{2.1}$$

Yukawa interactions couple up-type quarks to ϕ_1, and down-type quarks to either ϕ_1 or ϕ_2 (in order to avoid flavour changing neutral currents); it is irrelevant which one we choose, since we are going to neglect all couplings but the top's. The extra source of CP violation is provided by the angle ξ, which cannot be rotated away unless $\lambda_5 = \lambda_6$.

What we are really interested in, however, is the effective action, which includes thermal and quantum corrections. Extremizing the effective action, we can obtain the profile of the bubble wall; this computation is discussed in [8] (at least in the case $\lambda_1 = \lambda_2$, $v_1 = v_2$, in which the two neutral Higgses undergo a phase transition simultaneously). For the moment, however, we do not need an explicit solution, and write the VEV's in the general form:

$$< \phi_i^0(x) >= \rho_i(x) \exp(-i\theta_i(x)) \tag{2.2}$$

In the following, we shall use h_i^0, h_i^- ($i = 1, 2$) to refer to the four (complex) excitations in the Higgs fields.

An important quantity here is the width of the wall compared to the particle mean free path; the latter can be estimated using the cross section of the typical QCD process at this energy scale, i.e. gluon-mediated quark-quark scattering: as we shall show later, $\Gamma_{QCD} \sim T/3$. On the other hand, the wall width is strongly model-dependent, and even in the specific theory we are discussing it has not yet been computed precisely. For this reason, it would be useful to develop a formalism able to deal with both the "thin wall" and the "thick wall" limits, as well as the intermediate regime. Attempts in this direction will be discussed in the last section. Our work here applies, at least in its simplest form, to the thick wall limit, where the particles inside the wall can be taken to be in local kinetic equilibrium. Under those conditions, it is expedient to follow [9] and perform a hypercharge rotation of the fields:

$$f_i \to e^{2iy_i\theta_1(x)} f_i \tag{2.3}$$

where f_i is a generic field, and y_i its hypercharge.

After the rotation, the spacetime-dependent top mass becomes real (as we said, we are neglecting all other masses), but a new interaction term appears:

$$\mathcal{L}_{hyp} = -2\partial^\mu \theta J_\mu^Y \tag{2.4}$$

where J_μ^Y is the hypercharge current.

Our task is to study the effect of this term on the plasma density; let's first recapitulate the essential features of the system:

- The EW phase transition takes place at a temperature $T_c \sim 100GeV$. At this energy scale quarks and gluons are deconfined, and the strong coupling constant is small enough ($\alpha_s \sim 0.1$) for perturbation theory to be reliable.
- The Hubble expansion rate ($H \sim T^2/m_{pl}$) is much smaller than any other interaction rate, and can be ignored.
- We shall assume that each particle species i is locally in kinetic equilibrium, so that its density is determined by its chemical potential $\mu_i(x)$ and temperature.
- In principle, baryon production depends on the densities of all of the left-handed baryons and leptons. However, we shall ignore all leptons and the two lighter quark families, since their Yukawa couplings are much smaller than the top's.

With those assumptions, the system can be described in terms of just three quantum number densities:

$$q(x) \equiv t_L(x) + b_L(x) \tag{2.5}$$

$$t(x) \equiv t_R(x) \tag{2.6}$$

$$h(x) \equiv h_1^0(x) + h_1^-(x) + h_2^0(x) + h_2^-(x) \tag{2.7}$$

Below, we shall derive evolution (*i.e.*, transport) equations for the quantities (2.5)-(2.7), in the presence of the non-uniform external interaction (2.4). Unlike [9], which addressed the same problem, we compute the relevant parameters in terms of microscopic physics, using a general technique that can be applied to other (possibly more realistic) extensions of the Standard Model.

3 Transport equations

We refer to [6] for a derivation of macroscopic transport equations from microscopic principles; here we limit ourselves to presenting the final result and discussing its physical significance.

The key idea is that for quasiuniform systems (such as ours), it is possible to define local densities $n_i(\mathbf{k}, x^\mu)$ of particles with given on-shell momentum. The time evolution of these quantities can be derived by computing appropriate Green functions (*i.e.* Feynman diagrams) involving the elementary fields.

In the simple case of a scalar field with lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2} m^2 \phi^2(x) - V(\phi(x)) \tag{3.1}$$

we obtain

$$\frac{\partial n(\mathbf{k}, x^\mu)}{\partial t} + \mathbf{v} \cdot \nabla n(\mathbf{k}, x^\mu) + \frac{\partial \omega}{\partial x_\mu} \frac{\partial n(\mathbf{k}, x^\mu)}{\partial k^\mu} = W_e(1 + n(\mathbf{k}, x^\mu)) - W_a n(\mathbf{k}, x^\mu) \tag{3.2}$$

where ω is given by the space-time dependent dispersion relation, and W_e and W_a are the emission and absorption rates:

$$\begin{aligned} W_a(\mathbf{k}) &= \frac{1}{2\omega} \sum_{l,n} \left| < l | j(0) | n > \right|^2 \rho_{nn} (2\pi)^4 \delta^4(k - p_l + p_n) && (3.3) \\ W_e(\mathbf{k}) &= \frac{1}{2\omega} \sum_{l,n} \left| < n | j(0) | l > \right|^2 \rho_{ll} (2\pi)^4 \delta^4(k - p_l + p_n) && (3.4) \\ j(x) &\equiv -\frac{\delta V(\phi(x))}{\delta \phi(x)} && (3.5) \end{aligned}$$

Here ρ is the density matrix of the system, and the sums run over a complete set of states of the system. Eq. (3.2) applies to quasiuniform systems, but does not rely on the coupling constants being small, so that in principle we can compute W_a and W_e to any order in perturbation theory, provided that we use the correct finite-temperature Feynman rules (for a thorough discussion of the finite-temperature formalism, see [10]).

This procedure can be easily extended to more complicated cases, and to fermions; in the latter case, the statistical factors of $(1 + n)$ become $(1 - n)$. For each particle species we obtain an equation of the form (3.2) and every scattering process contributes a term to the W_e (W_a) of each emitted (absorbed) field.

In the case of kinetically thermalized particles, the reaction channel $A_1 ... A_n \to B_1 ... B_m \phi$ contributes to the functions W_a and W_e corresponding to ϕ as follows:

$$W_a(\mathbf{k}) = \frac{1}{2\omega} \int \prod_{A,B} \frac{d^3 p_i}{(2\pi)^3 2E_i} \left| j_{B\phi \to A} \right|^2 (2\pi)^4 \delta^4(k + P_B - P_A) \prod_B n_i \prod_A (1 + n_i) \tag{3.6}$$

$$W_e(\mathbf{k}) \;=\; \frac{1}{2\omega}\int \prod_{A.B} \frac{d^3p_i}{(2\pi)^3 2E_i} \left|j_{A\to B\phi}\right|^2 (2\pi)^4\delta^4(k-P_A+P_B)\prod_A n_i \prod_B (1+n_i) \quad (3.7)$$

so that the r.h.s. of Eq. (3.2) becomes:

$$\frac{1}{2\omega}\int \prod_{A,B} \frac{d^3p_i}{(2\pi)^3 2E_i} \left|j_{B\phi\to A}\right|^2 (2\pi)^4\delta^4(k+P_B-P_A)[n_\phi \prod_B n_i \prod_A (1+n_i) - (1+n_\phi)\prod_A n_i \prod_B (1+n_i)] \quad (3.8)$$

We thus recover the usual detailed balance conditions, and the (local) equilibrium density of a given particle species depends only on its dispersion relation.

Therefore, given a space-time dependent perturbation, the behaviour of the system is affected in two ways:

- Dispersion relations are modified, and that changes the local equilibrium densities;
- Source terms W_e and W_a appear, and they determine the rate at which local equilibrium is approached.

In order to find the total number density, we integrate Eq. (3.2) over $\mathbf{k}$. Elastic reactions (which do not change the identity of the involved particles, but only their momenta) give rise, after the integration is performed, to the usual diffusive behaviour. The interactions that do violate some particle quantum number, on the other hand, lead to net source terms (they contribute to diffusion as well, but we shall ignore this effect, because of its smallness).

In the next section, we shall apply Eq. (3.2) to our problem, considering both effects in detail, and presenting some explicit computations.

4 Results

The dispersion relation of a particle i is affected by the interaction $\mathcal{L}_{hyp}$ as follows:

$$k^\mu k_\mu = m_i^2 \to (k^\mu + 2y_i \partial^\mu \theta)^2 = m_i^2 \quad (4.1)$$

As mentioned above, this automatically provides values for the local equilibrium densities [9]:

$$f_{i,eq} = \frac{K_i \bar{\mu}_i T^2}{6} \tag{4.2}$$

where K_i = (number of spin degrees of freedom)$\times 2(1)$ for bosons (fermions), and $\bar{\mu}_i = -2\dot{\theta} y_i$.

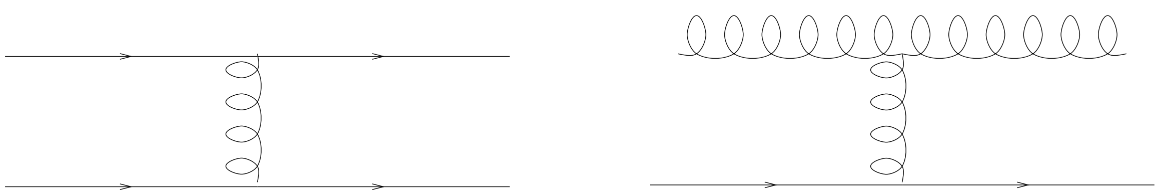
Assuming kinetic equilibrium, the local densities are of the form

$$n_i(\mathbf{k}) = \frac{e^{-\beta(E_i(\mathbf{k})-\mu_i)}}{1 \mp e^{-\beta(E_i(\mathbf{k})-\mu_i)}} \tag{4.3}$$

where $E_i = \sqrt{\mathbf{k}^2 + m_i^2} - \bar{\mu}_i$.

In the thick wall approximation, θ varies slowly, so that we can ignore the third term on the l.h.s. of Eq. (3.2), which involves the *second* derivatives of θ, and we only need compute the source terms W_e and W_a.

- **Quark elastic scattering** is dominated by QCD processes (see Fig.1).



(a) (b)

Figure 1: Dominant processes responsible for quark diffusion.

To lowest order, these processes can be computed assuming a perfectly thermalized system. In this case we do not have to use Eq. (3.2) and can instead apply standard finite-temperature field theory [10].

The finite-temperature rate of quark-scattering processes can be obtained (via the optical theorem) from the imaginary part of the quark self-energy. This computation is discussed in [11]; as emphasized there, diffusion depends not on the *total* cross section $\int d\sigma/d\Omega$, but on the *transport* cross section $\frac{3}{2}\int d\sigma/d\Omega \sin^2\theta$, where θ is the scattering angle (qualitatively, this is because close-to-forward scattering does not

contribute much to diffusion). Taking that into account, we obtain the interaction rate for a particle with momentum $\mathbf{p}$:[1]

$$\Gamma^{diff}_{quark}(\mathbf{p}) = \frac{9}{32\pi}\omega_{pl}^2 g^2 T C_F \int_{-1}^{1} d\cos\theta \int \frac{dk^2}{k^4 + \frac{9}{16}\omega_{pl}^4\cos^2\theta}\frac{k^2\sin^2\theta}{p^2+k^2-2pk\cos\theta} \quad (4.4)$$

where $p = |\mathbf{p}|$, $C_F = 4/3$, and $\omega_{pl}^2 = (C_V + N_f/2)g^2T^2/9$, with $C_V = 3$ and $N_f = 6$. The leading contribution comes from the infrared behaviour of the integrand, so we can write:

$$\Gamma^{diff}_{quark}(\mathbf{p}) \sim \frac{9}{32\pi}\omega_{pl}^2 g^2 T C_F \int_{-1}^{1} d\cos\theta \frac{1-\cos^2\theta}{p^2}\int \frac{dk^4}{2(k^4 + \frac{9}{16}\omega_{pl}^4\cos^2\theta)} \quad (4.5)$$

and, in the leading logarithm approximation,

$$\Gamma^{diff}_{quark}(\mathbf{p}) = \frac{2\pi}{3}\frac{T^3}{p^2}C_F(C_V + \frac{N_f}{2})\alpha_s^2 \ln\alpha_s^{-1} \sim 0.4\frac{T^3}{p^2} \sim \frac{T}{3} \quad (4.6)$$

(the last step was obtained by averaging over p).

- **Higgs elastic scattering** is dominated by electroweak processes (Fig.2):

  Diagram 2a is similar to those computed above, except that all of the particles here are bosons. A further difference is due to the vector boson mass, which provides an IR cutoff: the W and Z bosons receive mass not only from quantum corrections, but also from the coupling with the Higgs VEVs. However, both contributions to $m^2_{W,Z}$ are proportional to $g_W^2T^2$, and in the leading log approximation the precise coefficient is irrelevant. The $SU(2)_L$ and the $U(1)_Y$ sectors each contribute with a term:

$$\Gamma^{diff}_{Higgs}(\mathbf{p}) = \frac{4\pi}{3}\frac{T^3}{p^2}C_F(C_V + \frac{N_f}{2})\alpha^2\ln\alpha^{-1} \quad (4.7)$$

[1] $Im\Sigma^R$ in [11] is related to W_e and W_a by:

$$\begin{aligned} Im\Sigma^R &= -\Gamma\gamma^0/2; \\ \Gamma &= W_e(1+e^{\beta E}) \\ &= W_a(1+e^{-\beta E}) \end{aligned}$$

Figure 2: Dominant processes responsible for Higgs diffusion (wiggly lines stand for electroweak bosons, dashed lines for Higgses, and double lines for any particle coupled to electroweak bosons).

In the $SU(2)_L$ term $C_V = \frac{3}{4}$, $C_F = 2$, $\alpha = g^2/4\pi$ and N_f is the number of doublets coupled to the bosons; in the $U(1)_Y$ term $C_V = 0$, $C_F = 1$, $\alpha = g'^2/4\pi$ and $N_f = 2\sum y_i^2$, summed over all the hypercharged particles. The total diffusion width from Diagram 2a is

$$\Gamma^{diff}_{Higgs}(\mathbf{p}) \sim 0.15T \tag{4.8}$$

The Higgs being a boson, Diagram 2b has apparently the same IR behaviour as Diagram 2a, and has to be considered as well; however, in this case the exchanged particle has a mass $\sim O(\lambda T)$, where λ is the typical Higgs self-coupling constant (see Eq. (2.1)); if $\lambda \sim 1$, then the diagram contributes a subleading α^2 term, which we neglect.

Both for quarks and for Higgses, the diffusion constant is related to Γ by the usual formula:

$$D = \frac{1}{3\Gamma^{diff}} \tag{4.9}$$

- **top-quark mass coupling**, which mixes q and t (Fig.3a):

Strictly speaking, we cannot compute an interaction *rate* associated to such diagrams, because a quadratic perturbation leads to particle oscillations, not to exponential decay. However, it is still possible to compute an average rate (which does not vanish, since the particles experience thermal damping as well) as follows:

– As described in [11], the top quark retarded propagator in momentum space, including thermal correction, has the form:

$$G(p_\mu) = \frac{i\left[(p_0 - \Sigma_0)\,\gamma^0 - \mathbf{p}\cdot\gamma\left(1 - \frac{\Sigma_3}{p}\right)\right]}{(p_0 - \Sigma_0)^2 - p^2\left(1 - \frac{\Sigma_3}{p}\right)^2} \tag{4.10}$$

where $p = |\mathbf{p}|$ and Σ_0 and Σ_3 are functions of p_0 and p. The formula above is valid for massless fermions, and does not mix helicities.

– Adding the top mass introduces an off-diagonal term in the helicity basis:

$$G_{LL} = G_{RR} = \frac{i\left[(p_0 - \Sigma_0)\,\gamma^0 - \mathbf{p}\cdot\gamma\left(1 - \frac{\Sigma_3}{p}\right)\right]}{(p_0 - \Sigma_0)^2 - p^2\left(1 - \frac{\Sigma_3}{p}\right)^2 - m_t^2} \tag{4.11}$$

and

$$G_{LR} = G_{RL} = \frac{-im_t}{(p_0 - \Sigma_0)^2 - p^2\left(1 - \frac{\Sigma_3}{p}\right)^2 - m_t^2} \tag{4.12}$$

– We now Fourier transform G_{LR} with respect to time and obtain:

$$G_{LR}(t, \mathbf{p}) = -\frac{im_t}{\omega_p} e^{-\Gamma_b t/2} \sin(\omega_p t) \tag{4.13}$$

where $\omega_p^2 = p^2(1-\Sigma_3/p)^2 = p^2 + g_s^2 T^2 C_F/4$, and $\Gamma_b = 2Im\Sigma_0 = \alpha_s \pi^2 C_F T/6 \log(\alpha_s^{-1}) \sim 0.5T$ is the total scattering rate with the thermal bath.

– we can then compute the probability of finding a right top t_R at time $t + \Delta t$ coming from a left top at time t:

$$P(R \to L)(\Delta t) = |G_{LR}(\Delta t)|^2 = \frac{m_t^2}{\omega_p^2} e^{-\Gamma_b \Delta t} \sin^2(\omega_p \Delta t) \tag{4.14}$$

– finally, we can compute the rate of change of the right and left top quantum numbers; this is the sum of two terms, since to the time derivative of $P(R \to L)$ we must add the decay products of the left top itself, which have the same quantum number. We therefore obtain:

$$\begin{aligned} W_e(t_L \to t_R) &= \frac{m_t^2}{\omega_p^2} \frac{\int_0^\infty dt\ \left(\frac{d}{dt}\left(e^{-\Gamma_b t}\sin^2(\omega_p t)\right) + \Gamma_b\left(e^{-\Gamma_b t}\ sin^2(\omega_p t)\right)\right)}{\int_0^\infty dt\ e^{-\Gamma_b t}} t_L \\ &= \frac{2\Gamma_b m_t^2}{\Gamma_b^2 + 4\omega_p^2} t_L \end{aligned} \tag{4.15}$$

and, using Eq. (3.2):

$$\dot{t}_R(\mathbf{p}) = \frac{2m_t^2\Gamma_b}{(\Gamma_b^2 + 4\omega_p^2)}(t_L(\mathbf{p}) - t_R(\mathbf{p})) \tag{4.16}$$

In the limit of large Γ_b, this rate vanishes. As mentioned in [12], this happens because thermal interactions destroy quantum coherence, which is a necessary condition for these processes to occur.

Integrating over $\mathbf{p}$ we finally obtain, in terms of variables (7)-(9):

$$\dot{t} = \frac{m_t^2\Gamma_b}{32\pi^2T^2}(q - q_{eq} - 2(t - t_{eq})) \equiv \Gamma_{mass}\left(\frac{q}{6} - \frac{t}{3} - \frac{h_{eq}}{8}\right) \tag{4.17}$$

where q_{eq}, t_{eq}, and h_{eq} have been defined in Eq. (4.2).

- **Higgs mass matrix** (Fig.3b): the Higgs mass matrix should in principle be obtained from the quadratic term in the expansion of the effective action around the bubble solution; both outside and deep inside the bubble, the Higgs VEVs are slowly varying, and we can obtain the Higgs masses by expanding the finite temperature effective *potential*, $V_T(\phi)$ (we assume that the transition is first order, so in both regions $V_T(\phi)$ is convex and the masses are well defined). In the bubble wall, where the field configuration is more strongly space-time dependent, we have to expand the effective *action*. Unfortunately, we do not have an explicit expression for $V_T(\phi)$, nor for the effective action, so we shall just assume that the mass matrix elements are of order T. Higgs number violation is due to terms proportional to h_ih_j or $h_i^\dagger h_j^\dagger$, which are responsible for two point Green functions $G_{h_ih_j}$. Explicitly, the quadratic part of the neutral [2] Higgs self-coupling can be written (in Fourier space) as:

$$\mathcal{L}^{(2)} = \frac{1}{2}H^\dagger(\mathbf{k})\begin{pmatrix} k^2 - M & \Lambda \\ \Lambda^\dagger & k^2 - M \end{pmatrix} H(\mathbf{k}) \tag{4.18}$$

where

$$H = \begin{pmatrix} h_1^0 \\ h_2^0 \\ h_1^{0\dagger} \\ h_2^{0\dagger} \end{pmatrix} \tag{4.19}$$

[2]The electric charge is not spontaneously broken, so terms of the form $h_i^- h_j^-$ do not appear, and we can disregard charged Higgses altogether.

and M and Λ are 2×2-matrices.

Thermal interactions modify the dispersion relations by adding "mass-like" terms that are much smaller than the original masses, so we neglect this effect; thermal damping is taken into account by letting

$$k_0 \to \tilde{k}_0 \equiv k_0 + i\Gamma/2 \tag{4.20}$$

where Γ is the total Higgs scattering rate computed above.

Λ, which is responsible for Higgs number violation, will be taken to be small (in the opposite regime, we could content ourselves with the local equilibrium approximation); under that assumption, and in the basis that diagonalizes M, the propagators are:

$$|G_{h_i h_j}|^2 = e^{-\Gamma t}\sin^2\left(\frac{|\lambda_{ii}|t}{2\sqrt{\mathbf{k}^2+m_i^2}}\right)\delta_{ij} \tag{4.21}$$

and the same procedure as above yields to:

$$\dot{h} = -\Gamma^{(1)}_{Higgs}\frac{h-h_{eq}}{8} \tag{4.22}$$

where

$$\Gamma^{(1)}_{Higgs} = \frac{\Pi^2(|\lambda_{11}|+|\lambda_{22}|)}{4\zeta(3)\Gamma} \tag{4.23}$$

- **Higgs Yukawa couplings** (Fig.3c):

We shall assume that the Higgs that couples to the top is sufficiently light ($m_h < m_t - m_b$) or heavy ($m_h > m_t + m_b$), so that either $t_R \to b_L h^+$ or $h^- \to b_L \bar{t}_R$ can occur on shell; then, the leading contribution to $|j|^2$ is simply λ_t^2, with a normalization factor of $2m_i$ for each fermion.

In the first case, we obtain:

$$\begin{aligned}\dot{h} &= 2\lambda_t^2\int\frac{d\mathbf{k}_t}{(2\pi)^3 2E_t}\frac{d\mathbf{k}_h}{(2\pi)^3 2E_h}\frac{d\mathbf{k}_b}{(2\pi)^3 2E_b}\cdot 4m_t m_b(2\pi)^3\delta^3(\mathbf{k}_h+\mathbf{k}_b-\mathbf{k}_t) \\ &\quad 2\pi\delta\left(E_h - E_t + \sqrt{m_b^2+(\mathbf{k}_h-\mathbf{k}_t)^2}\right)[(n_h+1)(1-n_b)n_t - n_h n_b(1-n_t)]\end{aligned} \tag{4.24}$$

As we said, we assume kinetic equilibrium distributions, so:

$$\dot{h} = \frac{\lambda_t^2 m_t m_b}{4\pi^3} \int_{m_t}^{\infty} dE_t \int_{E_m}^{E_M} dE_h \frac{e^{-\beta E_t} \beta(-\mu_t - \mu_h + \mu_b)}{(1 - e^{-\beta(E_h + \mu_h)})(1 + e^{-\beta(E_b - \mu_b)})(1 + e^{-\beta(E_t - \mu_t)})} \tag{4.25}$$

where E_m and E_M are purely determined by the kinematics:

$$E_m = E_t \frac{m_t^2 + m_h^2 - m_b^2}{2m_t^2} - \sqrt{E_t^2 - m_t^2} \frac{\sqrt{(m_t^2 + m_h^2 - m_b^2)^2 - 4m_t^2 m_h^2}}{2m_t^2} \tag{4.26}$$

$$E_M = E_t \frac{m_t^2 + m_h^2 - m_b^2}{2m_t^2} + \sqrt{E_t^2 - m_t^2} \frac{\sqrt{(m_t^2 + m_h^2 - m_b^2)^2 - 4m_t^2 m_h^2}}{2m_t^2} \tag{4.27}$$

(in our case, we can safely ignore m_b, which is much smaller than the other masses)

Therefore:

$$\dot{h} = \dot{t} = -\dot{b} = -\frac{3\lambda_t^2 m_t m_b}{2\pi^3 T} \left(\frac{h}{8} - \frac{q}{6} + \frac{t}{3} \right) A \equiv -\Gamma_{yukawa} \left(\frac{h}{8} - \frac{q}{6} + \frac{t}{3} \right) \tag{4.28}$$

where

$$A(t \to bh^+) = \beta \int_{m_t}^{\infty} dE \frac{e^{\beta E}}{(1 + e^{\beta E})^2} \log \left(\frac{e^{\beta E_M} - 1}{e^{\beta E_m} - 1} \frac{e^{\beta E_m} + e^{\beta E}}{e^{\beta E_M} + e^{\beta E}} \right) \tag{4.29}$$

is of order 1.

In the second case, Eq. (4.28) still applies, with

$$A(h^- \to b\bar{t}) = \beta \int_{m_h}^{\infty} dE \frac{e^{\beta E}}{(1 - e^{\beta E})^2} \log \left(\frac{e^{\beta E_M} + 1}{e^{\beta E_m} + 1} \frac{e^{\beta E_m} + e^{\beta E}}{e^{\beta E_M} + e^{\beta E}} \right) \tag{4.30}$$

and the roles of m_t and m_h in E_M and E_m are interchanged.

If $h^0 \to t\bar{t}$ is also allowed (which is true at least in some region of the bubble wall, where the Higgs VEV, and consequently m_t, are sufficiently small), then we have another term, larger than (4.30) by a factor of m_t/m_b, which therefore dominates.

In the narrow window $m_t - m_b < m_h < m_t + m_b$, all these processes are kinematically forbidden and we must go to higher order, including QCD corrections, like

the one shown in Fig.4. The evaluation of this graph proceeds as in the previous computations; the exchanged particle is a fermion, so that both its propagator and its statistical factor make the diagram less IR divergent; the IR cutoff λ is provided by whichever is largest among the zero-temperature quark masses, their thermal corrections, and $|m_h - m_t|$. In any case, the result is suppressed, with respect to the previous case, by α_s:

$$\Gamma_{yukawa} \sim \frac{\lambda_t^2 \omega_{pl} |m_h - m_t|}{\Lambda T} \tag{4.31}$$

- **Higgs cubic coupling**: the effects of cubic Higgs couplings (Fig3d) depend on the Higgs masses; if at least one process of the form $h_i \to h_j h_k$ can occur on shell, its contribution can be evaluated as above (with the small modifications due to the different statistics):

$$\begin{aligned} \dot{h} &= \sum_{ijk} 2|\lambda_{ijk}|^2 \int \frac{d\mathbf{k}_i}{(2\pi)^3 2E_i} \frac{d\mathbf{k}_j}{(2\pi)^3 2E_j} \frac{d\mathbf{k}_k}{(2\pi)^3 2E_k} (2\pi)^3 \delta^3(\mathbf{k}_j + \mathbf{k}_k - \mathbf{k}_i) \cdot \\ & 2\pi\delta\left(E_j - E_i + \sqrt{m_k^2 + (\mathbf{k}_i - \mathbf{k}_j)^2}\right) [(n_j+1)(1+n_k)n_i - n_j n_k (1+n_i)] \end{aligned} \tag{4.32}$$

here the sum includes all combinations of Higgs and anti-Higgs fields that are kinematically allowed, and λ_{ijk} is a function of $\lambda_1 ... \lambda_6$ and of the Higgs VEVs.

We therefore obtain:

$$\dot{h} = -\sum_{ijk} \frac{3|\lambda_{ijk}|^2}{64\pi^2 T} \int_{m_i}^{\infty} dE \frac{e^{-\beta E}}{1-(e^{-\beta E})^2} \log\left(\frac{e^{\beta E_M}-1}{e^{\beta E_m}-1} \frac{e^{\beta E_m}-e^{\beta E}}{e^{\beta E_M}-e^{\beta E}}\right)(h - h_{eq}) \equiv -\Gamma^{(2)}_{Higgs} \frac{h-h_{eq}}{8} \tag{4.33}$$

The integral, formally divergent as $m_i \to 0$, is of order T if the Higgs masses are $\sim T$, as we expect.

If no 3-Higgs process is kinematically allowed, electroweak corrections must be included (Fig.4).

In this case,

$$\Gamma^{(2)}_{Higgs} = \Sigma_{ijk}\frac{|\lambda_{ijk}|^2}{T}\int \frac{d^4k}{(2\pi)^4}\delta\left[(p+k)^2 - m_j^2\right]\frac{Im\Pi(k)}{(k^2-m_k^2)^2}\coth(\frac{k_0}{2T}) \qquad (4.34)$$

Supposing that all the masses are $\sim T$, so that there is no IR divergence in the integral, and remembering that $Im\Pi \sim \alpha_{ew}T$, we obtain that

$$\Gamma^{(2)}_{Higgs} \sim \alpha_{ew}\Sigma_{ijk}\frac{|\lambda_{ijk}|^2}{T} \qquad (4.35)$$

with a coefficient of order 1.

Putting everything together, we obtain the following transport equations:

$$\begin{aligned}
\dot{q} &= D_{quark}\nabla^2 q - \Gamma_{yukawa}\left(\frac{q}{6}-\frac{h}{8}-\frac{t}{3}\right) - \Gamma_{mass}\left(\frac{q}{6}-\frac{h_{eq}}{8}-\frac{t}{3}\right) && (4.36)\\
\dot{t} &= D_{quark}\nabla^2 q + \Gamma_{yukawa}\left(\frac{q}{6}-\frac{h}{8}-\frac{t}{3}\right) + \Gamma_{mass}\left(\frac{q}{6}-\frac{h_{eq}}{8}-\frac{t}{3}\right) && (4.37)\\
\dot{h} &= D_{Higgs}\nabla^2 h + \Gamma_{yukawa}\left(\frac{q}{6}-\frac{h}{8}-\frac{t}{3}\right) - \Gamma_{Higgs}\left(\frac{h-h_{eq}}{8}\right) && (4.38)
\end{aligned}$$

where D_{quark}, D_{Higgs}, Γ_{yukawa}, $\Gamma_{Higgs} \equiv \Gamma^{(1)}_{Higgs} + \Gamma^{(2)}_{Higgs}$ are given in Eqs. (4.6), (4.7), (4.17), (4.28), (4.23), (4.33).

5 Discussion and comparison with previous work

Eqs. (4.36)-(4.38) were first derived in [9], who interpreted $\mathcal{L}_{hyp}$ as an effective chemical potential and used elementary thermodynamics. They made reasonable assumptions to estimate the Γ and D coefficients, as well as both weak and strong sphaleron effects, without explicitly computing any of these quantities. In our approach, sphaleron contributions, which cannot be computed in perturbation theory and have been ignored in all our discussion, have still to be introduced by hand. However, they only introduce additional terms in Eqs. (4.36)-(4.38), without affecting the terms we did compute.

One might still wonder whether such a detailed computation is anything more than an academic exercise, given that there is no compelling evidence to support this specific model, let alone to constrain its many parameters. Nevertheless, in the course of our work we have reached a few conclusions that are worth pointing out:

- **Generalizations**: the technique we have used can be applied to any other model; admittedly, the thick wall approximation has simplified our task, but it can be relaxed: Eq. (3.2) is valid if the wall is thicker than $1/T$, a condition considerably weaker than the usual "thick wall" assumption, which requires $L_w >> \tau \sim 1/\alpha_s^2 T$. If the latter constraint is not satisfied, but the first one is, we can still use Eq. (3.2), but we cannot assume local kinetic equilibrium. This means that the local densities are not simply parametrized by chemical potentials $\mu_i(x)$, and the whole computation becomes quite cumbersome, but hopefully still tractable.

  We should mention here that [12] tried to avoid this complication, deriving a unified formalism that does not require the wall to be thin or thick. They derived transport equations similar to ours, by adding source terms (corresponding to our h_{eq},t_{eq},q_{eq} terms) to the unperturbed evolution. The sources were computed for generic wall thickness, but taking only quadratic interactions into account; for this reason, they found vanishing effects in the thick wall limit, at odds with our own result. For realistic values of L_w the numerical disagreement may not be important, but still it is not negligible.

- **Diffusion**: The quark and Higgs diffusion constants that we obtained are noticeably smaller than the estimates in [13]: $D_{quark} = 6/T$, $D_{Higgs} = 110/T$, whereas they are comparable with the values used by [9] for their numerical analysis: $D_{quark} = 3/T$, $D_{Higgs} = 10/T$; for this reason we believe that the qualitative results and the discussion contained in [9] are still valid.

- **Source terms:** in [9], Γ_{Higgs} and Γ_{mass} were estimated to be:

$$\Gamma_{Higgs} \sim \Gamma_{mass} \sim \frac{m_t^2}{T} \tag{5.1}$$

  As we said earlier, Γ_{mass} is sensitive to the quark mean free path, and the value we obtained in Eq. (4.17) is considerably smaller. Γ_{Higgs} strongly depends on the unknown parameters of the Higgs sector; Eq. (4.33) can be used to study the whole parameter space, but realistically we won't obtain results that are too different from (5.1).

- **Baryon number**: [9] have integrated Eq. (4.36)-(4.38) numerically, and found that values of n_B/s consistent with experimental estimates can be obtained in this model by taking $\xi \sim 10^{-2} - 10^{-3}$. We have not performed the same calculation with our values of the Γ's and the D's, but we believe that their conclusions would not be qualitatively altered. Therefore, this model should still be regarded as a viable candidate for electroweak baryogenesis.

Acknowledgements

It is a pleasure to thank Andrew Cohen for suggesting this project and for his insightful assistance and patient encouragement. This work was supported in part under grant DE-FG02-91ER40676.

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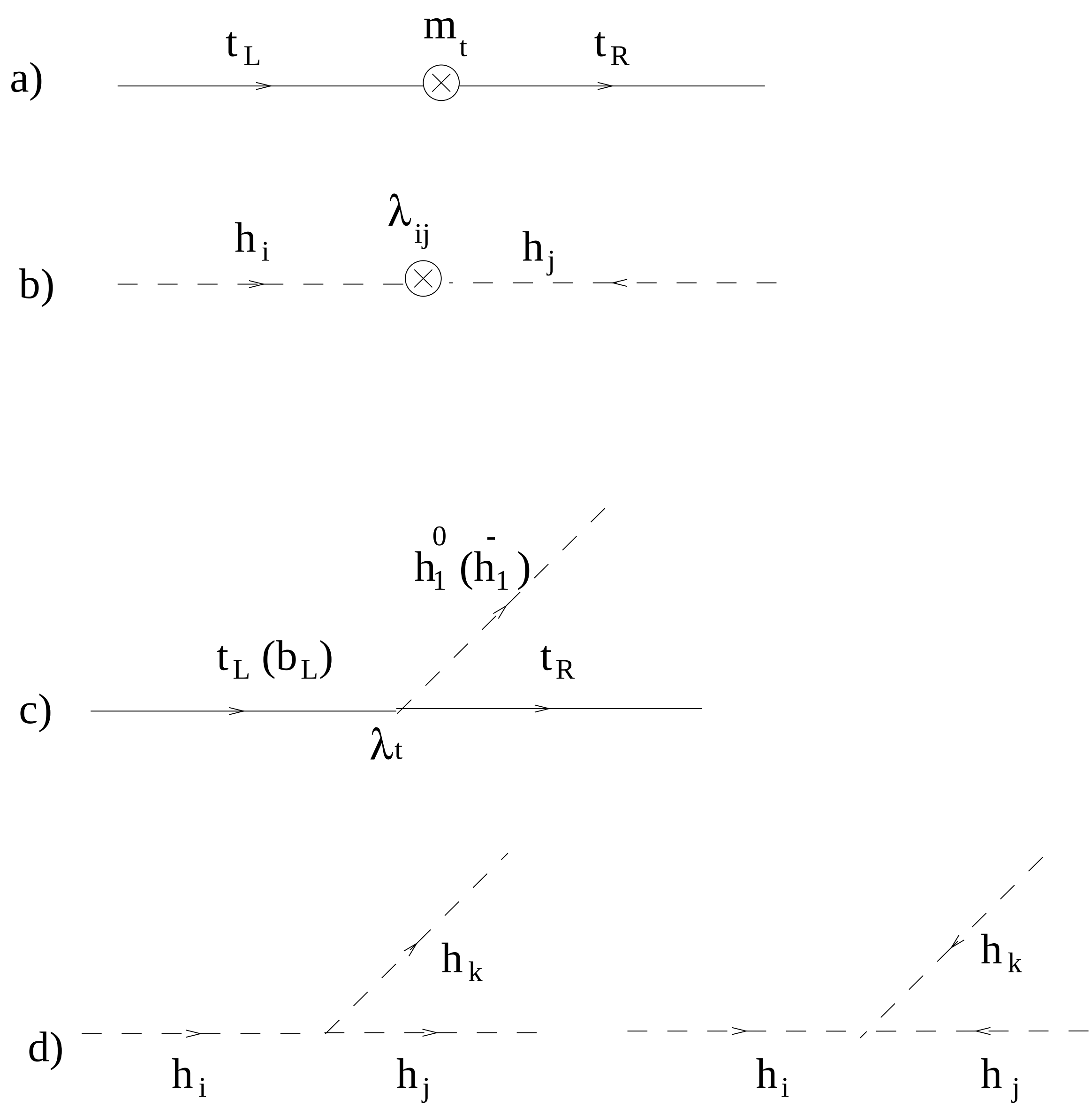

Figure 3: Vertices that violate at least one among quantum numbers t, q, h.

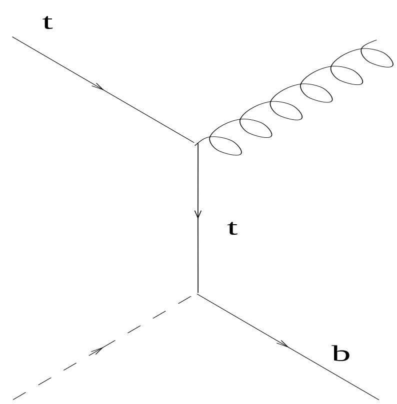

Figure 4:

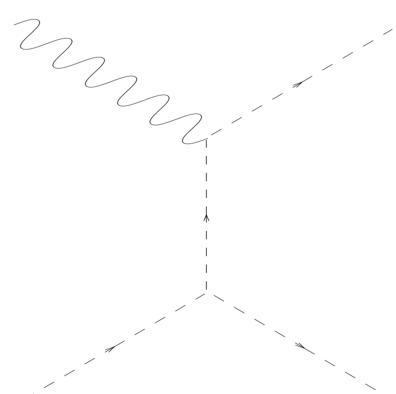

Figure 5: